

# Predictive Avoidance for Ground-Based Laser Illumination

S. Alfano\*

*U.S. Space Command, Peterson Air Force Base, Colorado 80914*  
 and

R. Burns,<sup>†</sup> D. Pohlen,<sup>‡</sup> and G. Wirsig<sup>‡</sup>

*U.S. Air Force Research Laboratory, Kirtland Air Force Base, New Mexico 87117*

**Predictive avoidance determines discrete windows of time for safe laser illumination from a ground-based laser platform. That is, the analysis identifies time periods during which the laser will not inadvertently illuminate a satellite. The geometry of the problem is utilized to filter satellites that cannot possibly be illuminated by a specific laser platform. Further, several nongeometric filters are introduced to filter remaining satellites by removing those which cannot be illuminated by the platform in a specified period. Both sets of filters improve the efficiency of the analysis by reducing the number of state propagations necessary. A method for determining the times of possible accidental illumination using cubic polynomial approximation is also presented. Finally, a test case is run that considers the entire catalog of objects publicly available from the U.S. Space Command. The complete analysis, which considers the entire catalog as potential victims for one target satellite pass, yields verified results in less than 3.5 s on a laptop personal computer running at 266 MHz.**

## Nomenclature

$c(\tau)$	= cubic spline
$e_{low}$	= minimum topocentric elevation angle at the laser platform
$f(t)$	= function that indicates whether potential victim is in safety cone
$i_{pv}$	= inclination of the potential victim
$p$	= semilatus rectum
$r$	= distance from center of the Earth to edge of the engagement envelope
$r_a$	= geocentric apogee distance
$r_p$	= geocentric perigee distance
$\mathbf{r}_{pl}$	= geocentric position vector of the laser platform
$\mathbf{r}_{pv/s}$	= position vector of potential victim relative to the laser platform
$\mathbf{r}_{sat}$	= position vector for satellite
$\mathbf{r}_{t/s}$	= position vector of target relative to the laser platform
$t_{min}$	= actual time of minimum $f(t)$
$t_{new}$	= estimate of time of function $f(t)$ minimum based on $c(\tau)$
$t_{start}$	= time at beginning of illumination
$t_{stop}$	= time at end of illumination
$(t_1, t_2)$	= time interval input for cubic spline
$\mathbf{v}_{pv/s}$	= velocity vector of potential victim relative to the laser platform
$\mathbf{v}_{sat}$	= velocity vector for satellite
$\mathbf{v}_{t/s}$	= velocity vector of target relative to the laser platform
$\alpha$	= separation angle between potential victim and target
$\beta$	= safety cone half-angle
$\gamma_i$	= order $i$ coefficient
$\Delta\phi$	= maximum geocentric angle between laser platform and potential victim such that it enters the engagement envelope at apogee
$\Delta\phi_{best}$	= geocentric angle between laser platform and orbital plane of potential victim
$\Delta\phi_{max}$	= geocentric angle between laser platform and the edge of the engagement envelope
$\eta$	= angle opposite $\Delta\phi$ in apogee filter

$\theta_{pl}$	= platform latitude
$\mu$	= gravitational parameter of the Earth
$\rho_{max}$	= maximum laser range
$\rho_p$	= platform to satellite distance when the satellite is at perigee and at the platform's longitude
$\omega_p$	= angular rate of satellite at perigee
$\omega_{\oplus}$	= rotation rate of the Earth

## Introduction

WHEN a high-energy laser emits a beam into the sky for any purpose (such as satellite ranging), a predictive avoidance analysis must be performed to ensure that no satellites are inadvertently illuminated. Inadvertent illumination of satellites with high-energy lasers can cause serious damage to satellites, especially those satellites carrying sensors sensitive to the laser wavelength. The most straightforward predictive avoidance methods require lengthy propagation and comparison of every object in the satellite catalog to the moving target vector at small time intervals throughout the period of interest. In an effort to accelerate this procedure, methods to filter the satellites based on geometry and location are presented. Satellites that meet the criteria of the individual filters are eliminated from further consideration, and the remaining candidate satellites are analyzed to determine whether inadvertent illumination may occur. In this latter analysis, cubic polynomials are utilized as interpolation tools to allow larger time steps than a simple, but time consuming, brute force propagation method. The result is a list of satellites and associated times when they will pass within a specified safety cone. These times are refined to within a user-specified precision using the Newton–Raphson root finding algorithm.

The analysis uses the following as inputs: target ephemeris, the catalog of satellite elements of appropriate date, platform coordinates, laser range and lowest allowable pointing angle, and the time and length of the illumination period. The software that has been developed to test and evaluate this algorithm is written in Pascal and assumes that the laser is firing at some target. The software can easily be modified to accommodate a laser firing at a fixed azimuth and elevation or any other prescribed trajectory.

This method processes each satellite, called a potential victim, through a series of six filters to determine whether the laser poses any threat of accidental illumination. The filters are sequential; failing a filter, the potential victim is removed from further consideration. The first three filters are dependent only on the geometry of the problem. That is, they do not require the solution of the differential equations of motion; therefore, they can be executed with very little computational expense. Filters 4 and 5 require knowledge of the

Received 24 November 1998; revision received 1 April 1999; accepted for publication 28 April 1999. This material is declared a work of the U.S. Government and is not subject to copyright protection in the United States.

\*Vice Director, Analysis.

<sup>†</sup>Engineer, Space Vehicles Directorate. Member AIAA.

<sup>‡</sup>Engineer, Space Vehicles Directorate.

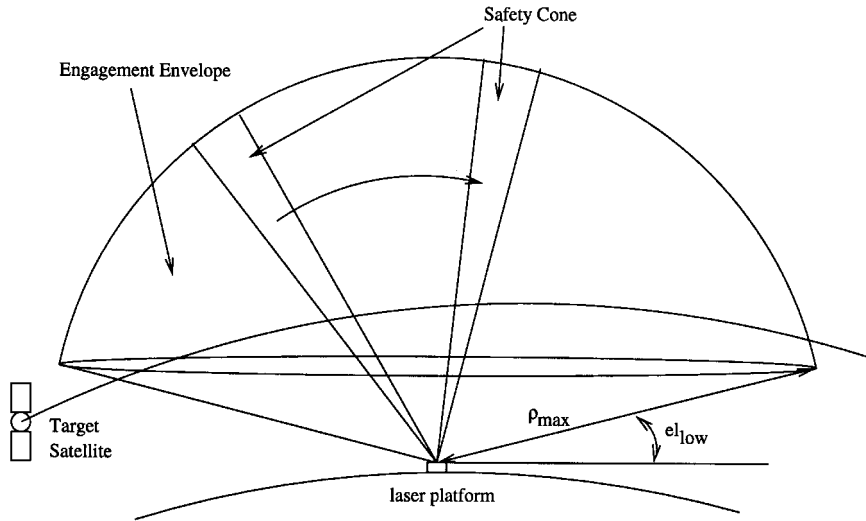


Fig. 1 Platform geometry.

potential victim position and velocity at the beginning of the illumination period; this information is spatially related to the largest illumination cone that can be swept out by the laser. Filter 6 requires the determination of the state vector for the potential victim at the end of the illumination. The final analysis evaluates the angle of separation between the satellite and target throughout the illumination pass and determines when the angle is within a specified safety cone angle. That is, a time history is needed of the separation angle  $\alpha$ , between target and potential victim, subtended at the platform. Cubic polynomials are used to approximate a function related to  $\alpha$  to mitigate the number of function evaluations needed to determine whether and when a particular potential victim enters the safety cone. Although not addressed in this work, the problem can be further reduced by filtering out known space debris, dead satellites, etc., which account for a substantial percentage of the satellite catalog.

### Preprocessing

Several parameters must be defined prior to running a potential victim through the gauntlet of filters. The most obvious is the position of the target at every moment during the illumination pass. This target ephemeris, coupled with the platform latitude, longitude, and height above sea level, should be enough to define the laser position and direction throughout the pass.

Additionally, two geometrical volumes are defined, the engagement envelope and the safety cone (Fig. 1). The engagement envelope is defined, in part, by the minimum topocentric elevation angle  $e_{low}$  for the target pass. If  $e_{low}$  is unknown, the lowest angle physically possible from the platform can be used. This angle, along with the maximum effective range of the laser  $\rho_{max}$ , defines a capped cone called the engagement envelope emanating from the platform. In a similar manner, the safety cone is defined from  $\rho_{max}$  and the minimum allowed angle  $\alpha$  of separation between target and potential victim. Unlike the engagement envelope, the safety cone moves with the platform-target vector  $r_{t/s}$  throughout the pass. Finally, the start and end times complete the information needed to define the target pass parameters.

At this point, each satellite can be characterized for evaluation by the filters. A shell volume in the space about the Earth can be defined for a satellite based on its inclination  $i_{pv}$ , perigee radius  $r_p$ , and apogee radius  $r_a$ . The intersection of the engagement envelope and this volume are used in filters 1–3.

### Geometric Filters

The first three filters do not require the determination of any satellite position; they rely solely on the geometry of the problem. That is, the potential victim can be filtered based on the combination of the platform coordinates and the potential victim's elements, which are read from the satellite catalog. The essence of these filters follows the filtering scheme presented by Alfano and Negron.<sup>1</sup>

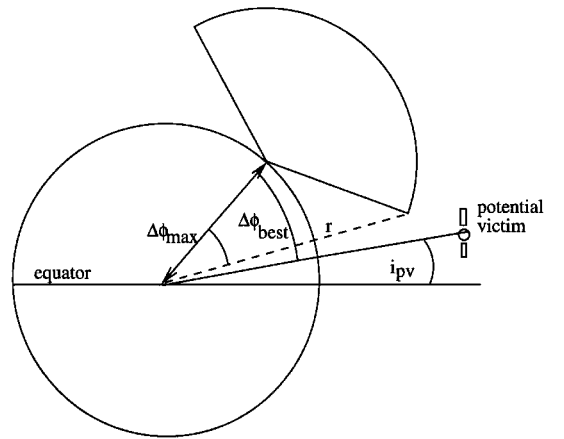


Fig. 2 Inclination filter.

#### Filter 1: Inclination

This filter determines whether it is geometrically possible for a potential victim to exceed the laser's minimum elevation angle (Fig. 2). We calculate the smallest possible geocentric angular distance between platform and victim,  $\Delta\phi_{best}$ , which is found by subtracting the maximum latitude (inclination) of the potential victim from the platform latitude. If the victim's orbit surpasses the platform latitude,  $\Delta\phi_{best}$  is set to zero, signifying that at some point the satellite could fly directly overhead. That is,

$$\Delta\phi_{best} = \begin{cases} \theta_{pl} - i_{pv} & \text{if } \theta_{pl} > i_{pv} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

Once  $\Delta\phi_{best}$  is determined, we want to compare it to  $\Delta\phi_{max}$ , which represents the maximum geocentric angle reached by the laser based on  $\rho_{max}$  and  $e_{low}$ . From the law of cosines,  $\Delta\phi_{max}$  is found using

$$\Delta\phi_{max} = \cos^{-1} \left( \frac{r_{pl}^2 + r^2 - \rho_{max}^2}{2r_{pl}r} \right) \quad (2)$$

where  $r$  is also calculated from the law of cosines:

$$r^2 = r_{pl}^2 + \rho_{max}^2 + 2r_{pl}\rho_{max} \sin e_{low}$$

Note that  $\Delta\phi_{max}$  only needs to be calculated once per analysis because it depends only on the platform coordinates and laser parameters. If  $\Delta\phi_{best} > \Delta\phi_{max}$  we conclude that the victim will never enter the engagement envelope and will not be in danger of illumination. This filter becomes more effective the further a platform is from the equator and the smaller the laser range.

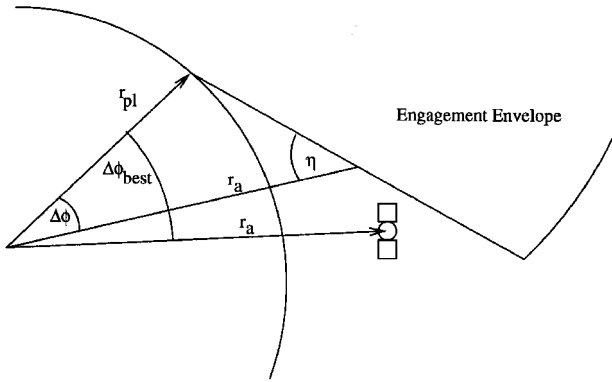


Fig. 3 Apogee filter.

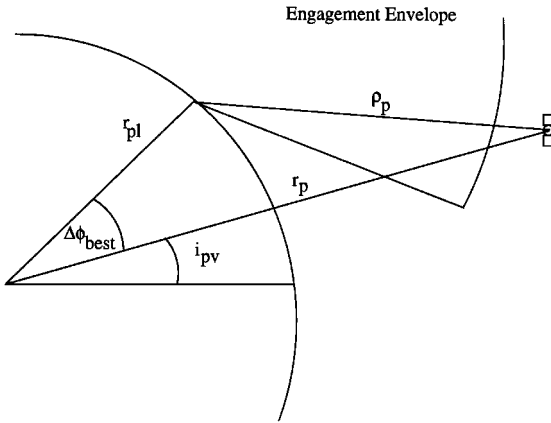


Fig. 4 Perigee filter.

**Filter 2: Apogee**

This filter determines if a potential victim's apogee radius  $r_a$  will always place the orbit below the engagement envelope at  $\Delta\phi_{\text{best}}$  (Fig. 3). The law of sines is used to find the largest geocentric angle  $\Delta\phi$  that will allow the potential victim to penetrate the engagement envelope:

$$\eta = \sin^{-1}\left(\frac{r_{\text{pl}} \cos e_{\text{low}}}{r_a}\right) \quad (3)$$

Summing the angles in the triangle yields

$$\Delta\phi = [(\pi/2) + e_{\text{low}}] - \eta \quad (4)$$

Substituting Eq. (3) into Eq. (4) gives the value for  $\Delta\phi$ , which is compared to  $\Delta\phi_{\text{best}}$ . If  $\Delta\phi < \Delta\phi_{\text{best}}$ , then the potential victim will never enter the engagement envelope, and it is removed from the further analysis. Note that this filter is designed to accommodate negative  $e_{\text{low}}$  values.

**Filter 3: Perigee**

This filter is similar to the apogee filter; it determines if a potential victim's perigee radius  $r_p$  will always place the orbit above the engagement envelope at  $\Delta\phi_{\text{best}}$ . Again using the law of cosines, the range from the platform to the potential victim at perigee  $\rho_p$  is determined when the geocentric angle is at  $\Delta\phi_{\text{best}}$  (Fig. 4):

$$\rho_p^2 = r_p^2 + r_{\text{pl}}^2 - 2r_p r_{\text{pl}} \cos \Delta\phi_{\text{best}} \quad (5)$$

If  $\rho_p > \rho_{\text{max}}$ , then the potential victim will never enter the engagement envelope, and it is removed from the further analysis. Of course, the effectiveness of this filter increases for decreasing values of  $\rho_{\text{max}}$ , the maximum range of the laser.

**Nongeometric Filters**

The final two of the remaining filters make the assumption that the maximum illumination period will be 15 min. This assumption

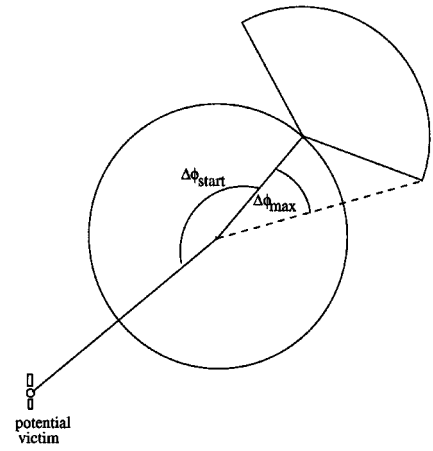


Fig. 5 Too-far-away filter.

enables the algorithm to eliminate potential victims that cannot enter the engagement envelope during the illumination period. For a conservative estimate of the minimum time necessary for any satellite to complete one-quarter of an orbit, 15 min was chosen. The reason for restricting the analysis to the time for the fastest possible satellite to complete one-quarter of an orbit will become evident in the discussion of the individual filters. Limiting the length of the illumination period is justified because, for longer periods of interest, multiple analyses may be performed for each 15-min block within the period.

**Filter 4: Too Far Away**

If a potential victim will be outside the engagement envelope during the entire period of illumination, then it can be eliminated for that pass. This filter requires an orbit propagator to determine the satellite's position and velocity vectors ( $\mathbf{r}_{\text{sat}}$  and  $\mathbf{v}_{\text{sat}}$ , respectively) at the lasing start time  $t_{\text{start}}$ . By using the angular rate of the satellite at perigee,

$$\omega_p = \sqrt{\mu p}/r_p^2 \quad (6)$$

the platform vector  $\mathbf{r}_{\text{pl}}$ , and the Earth's rotational rate  $\omega_{\oplus}$ , we can make a conservative estimate to determine whether the geocentric angle between platform and satellite will be large enough so that the satellite could not enter the engagement envelope during the lasing period (Fig. 5).

The required angles are calculated using

$$\Delta\phi_{\text{start}} = \cos^{-1}\left(\frac{\mathbf{r}_{\text{sat}} \cdot \mathbf{r}_{\text{pl}}}{|\mathbf{r}_{\text{sat}}||\mathbf{r}_{\text{pl}}|}\right) \quad (7)$$

$$\Delta\phi_{\text{avoid}} = \Delta\phi_{\text{max}} + (\omega_p + \omega_{\oplus})\Delta t \quad (8)$$

where  $\Delta t$  is the duration of lasing. If  $\Delta\phi_{\text{start}} > \Delta\phi_{\text{avoid}}$ , then the satellite will not be at risk.

**Filter 5: Moving Away**

This filter builds on the concept of filter 4 by eliminating those satellites outside the engagement envelope at  $t_{\text{start}}$ , that is,  $\Delta\phi_{\text{start}} > \Delta\phi_{\text{max}}$ , and moving away (Fig. 6). Moving away can be defined as

$$(\mathbf{r}_{\text{sat}} - \mathbf{r}_{\text{pl}}) \cdot \mathbf{v}_{\text{sat}}^{\text{ECEF}} < 0 \quad (9)$$

where  $\mathbf{v}_{\text{sat}}^{\text{ECEF}}$  is the velocity of the potential victim satellite in the Earth centered Earth fixed rotating reference frame. Here the assumption that no satellite can complete more than one-quarter orbit during the period of interest is critical to the efficacy of this filter. That is, if a satellite is outside the engagement envelope and moving away at the beginning of the period, then it cannot possibly reenter the engagement envelope during this period.

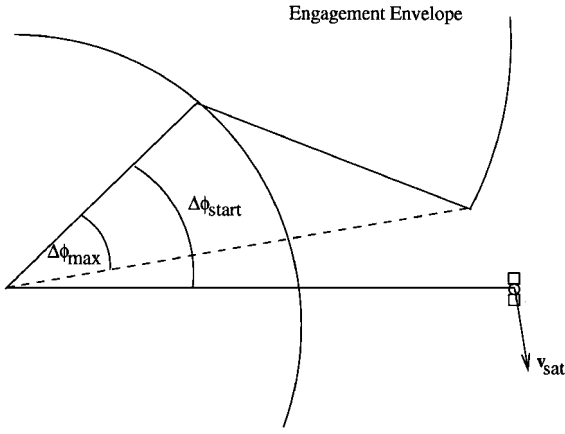


Fig. 6 Moving-away filter.

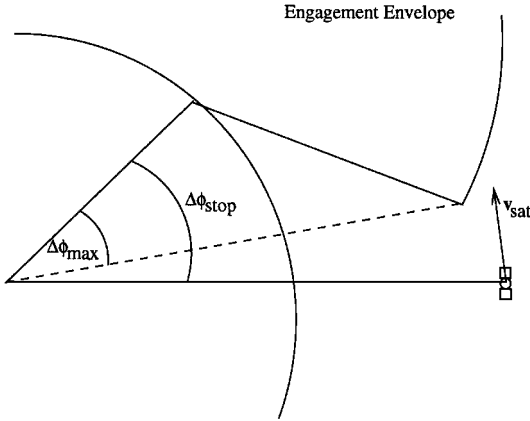


Fig. 7 Moving-toward filter.

**Filter 6: Moving Toward**

Similar to filter 5, this filter eliminates those satellites outside the engagement envelope at laser stop time  $t_{\text{stop}}$ , that is,  $\Delta\phi_{\text{stop}} > \Delta\phi_{\text{max}}$ , and moving toward (Fig. 7). As in Eq. (9), moving toward can be defined as

$$(\mathbf{r}_{\text{sat}} - \mathbf{r}_{\text{pl}}) \cdot \mathbf{v}_{\text{sat}}^{\text{ECEF}} < 0 \quad (10)$$

This test requires propagating platform and potential victim to  $t_{\text{stop}}$ . As with the preceding filter, the restriction of the illumination period to 15 min allows this filter to eliminate satellites that fulfill the preceding criteria because they must have been outside the engagement envelope for the entire period.

**Safety Cone Analysis**

The remaining potential victims must be analyzed to determine entry and exit times of any penetrations of the safety cone. The safety cone, with prescribed half-angle  $\beta$ , moves such that its axis of symmetry lies on the position vector  $\mathbf{r}_{t/s}$  of the target satellite relative to the platform. Hence, by determining the angle  $\alpha$  between the position vector  $\mathbf{r}_{pv/s}$  of the potential victim relative to the platform and  $\mathbf{r}_{t/s}$  using

$$\cos \alpha = \frac{\mathbf{r}_{pv/s} \cdot \mathbf{r}_{t/s}}{|\mathbf{r}_{pv/s}| |\mathbf{r}_{t/s}|} \quad (11)$$

all possible safety cone penetrations as well as the associated entry and exit times can be found (Fig. 8). However, it is more computationally expedient to define a function that can be approximated by polynomials over the time interval between points where the function has been evaluated. The most straightforward function

$$g(t) = \cos \alpha - \cos \beta \quad (12)$$

may lead to problems discerning the roots of  $g(t) = 0$  at  $\alpha = \beta$ . Because  $\beta$  is generally small, the time derivative  $\dot{g}(t)$  is nearly zero near the roots because the derivative of  $\cos \alpha$  is nearly zero close

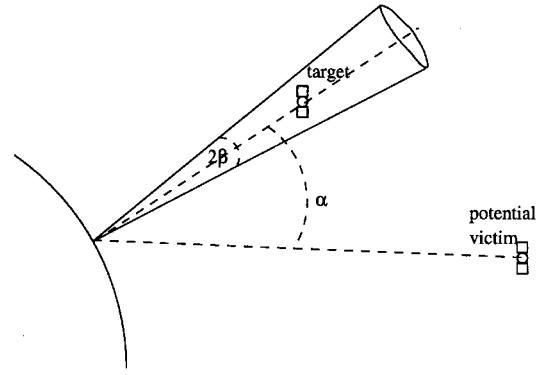


Fig. 8 Separation angle.

to  $\alpha = 0$ . Thus, the function will cross the  $x$  axis in a very shallow manner making the roots difficult to determine to a prescribed precision. Instead, consider the function

$$h(t) = \sin \alpha - \sin \beta \quad (13)$$

which crosses the  $x$  axis in a sharper fashion. Alternately, this function can be written

$$h(t) = \frac{|\mathbf{r}_{pv/s} \times \mathbf{r}_{t/s}|}{|\mathbf{r}_{pv/s}| |\mathbf{r}_{t/s}|} - \sin \beta \quad (14)$$

To simplify the time derivative of this function, multiply through by the denominator of the first term, yielding

$$f(t) = |\mathbf{r}_{pv/s} \times \mathbf{r}_{t/s}| - |\mathbf{r}_{pv/s}| |\mathbf{r}_{t/s}| \sin \beta \quad (15)$$

which has the same roots as  $h(t)$  and time derivative

$$\dot{f}(t) = \frac{[(\mathbf{v}_{pv/s} \times \mathbf{r}_{t/s}) + (\mathbf{r}_{pv/s} \times \mathbf{v}_{t/s})] \cdot (\mathbf{r}_{pv/s} \times \mathbf{r}_{t/s})}{|\mathbf{r}_{pv/s} \times \mathbf{r}_{t/s}|} - \left\{ (\mathbf{v}_{pv/s} \cdot \mathbf{r}_{t/s}) \frac{|\mathbf{r}_{t/s}|}{|\mathbf{r}_{pv/s}|} + (\mathbf{r}_{pv/s} \cdot \mathbf{v}_{t/s}) \frac{|\mathbf{r}_{pv/s}|}{|\mathbf{r}_{t/s}|} \right\} \sin \beta \quad (16)$$

The roots of  $f(t)$  represent entry and exit times for the potential victim's intersection with the safety cone. In other words,  $f(t)$  is positive when the potential victim is outside the safety cone and negative when the potential victim is inside the cone.

**Interpolation**

Rather than evaluate  $f(t)$  at small time increments throughout the period of interest, it is desirable to use interpolation to reduce the number of function evaluations and, consequently, the number of propagation calls. Cubic polynomial splines are used to approximate  $f(t)$  in a method adapted from Alfano.<sup>2</sup> That is, given the value of  $f(t)$  and  $\dot{f}(t)$  at  $t_n$  and  $t_{n+1}$ , the cubic polynomial is defined as

$$c(\tau) = \gamma_3 \tau^3 + \gamma_2 \tau^2 + \gamma_1 \tau + \gamma_0 \quad (0 \leq \tau \leq 1) \quad (17)$$

where

$$\gamma_0 = f(t_n), \quad \gamma_1 = \dot{f}(t_n) \Delta t$$

$$\gamma_2 = -3f(t_n) - 2\dot{f}(t_n) \Delta t + 3f(t_{n+1}) - \dot{f}(t_{n+1}) \Delta t$$

$$\gamma_3 = 2\dot{f}(t_n) + \dot{f}(t_n) \Delta t - 2\dot{f}(t_{n+1}) + \dot{f}(t_{n+1}) \Delta t$$

$$\Delta t = t_{n+1} - t_n \quad \text{for } t_{n+1} \geq t_n$$

$$\tau = (t - t_n) / \Delta t \quad \text{for } t_n \leq t \leq t_{n+1}$$

Note that this cubic spline matches the values of  $f$  and  $\dot{f}$  at  $t_n$  and  $t_{n+1}$ .

Evaluating the function at regular intervals throughout the illumination period and using the spline technique between them approximate the function very well over most of the period. However, the spline tends to stray from the function near its minimum if none

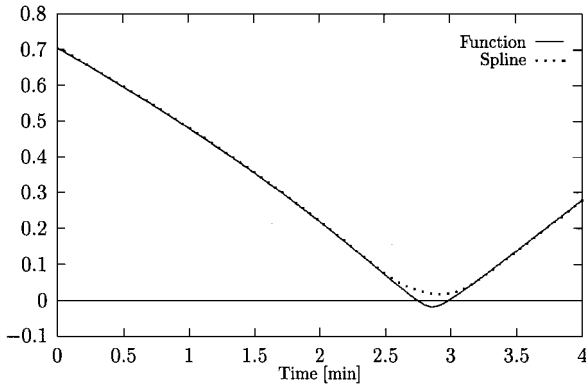


Fig. 9 Spline separates from the function near its minimum.

of the evaluations are close to the minimum (Fig. 9). Alternately, the state vectors at the endpoints of the illumination period ( $t_{\text{start}}$  and  $t_{\text{stop}}$ ), which have been previously calculated for filters 4–6, are used to evaluate  $f(t)$  and  $\dot{f}(t)$  to determine one cubic polynomial approximating the function over the entire interval. The analysis uses a recursive algorithm to determine if the potential victim enters the safety cone. If it does, the algorithm determines the entry and exit times. The initial estimates for the entry and exit times are then refined to a user-prescribed precision using the Newton–Raphson technique.<sup>3</sup> The algorithm uses the spline to approximate the time  $t_{\text{new}}$  when the function is a minimum. Evaluating  $f(t_{\text{new}})$  and  $\dot{f}(t_{\text{new}})$ , the algorithm eliminates intervals that do not contain roots and reenters the algorithm with a smaller time interval.

#### Algorithm

- 1) Input time interval  $(t_1, t_2)$  and the values  $f(t_1)$ ,  $f(t_2)$ ,  $\dot{f}(t_1)$ , and  $\dot{f}(t_2)$ .
- 2) Calculate coefficients  $\gamma_i$  of cubic spline  $c(\tau)$ .
- 3) If  $f < 0$  at both endpoints, then the potential victim remains in the safety cone for the entire period. End algorithm.
- 4) If  $f < 0$  at one of the endpoints, then that time is taken as the entry or exit time and the cubic is solved for the root between  $t_1$  and  $t_2$ . End algorithm.
- 5) If  $f > 0$  at both endpoints, then the solution of  $c'(\tau) = 0$  yields  $t_{\text{new}}$  as an approximation of the time  $t_{\text{min}}$  that the function is at a minimum.
- 6) Evaluate  $f(t_{\text{new}})$ .
  - a) If  $f(t_{\text{new}}) < 0$ , then  $t_{\text{new}}$  is considered close enough to  $t_{\text{min}}$  to yield an accurate representation of the function around the minimum. Solve  $c(\tau) = 0$  for  $t_{\text{root1}}$  and  $t_{\text{root2}}$ . End algorithm.
  - b) If  $f(t_{\text{new}}) > 0$ , then proceed as follows. i) If  $f(t_{\text{new}}) - \dot{f}(t_{\text{new}})\Delta t > 0$ , then the function cannot cross the  $x$  axis due to its concave nature near the minimum. The potential victim does not penetrate the safety cone. End algorithm. However, ii) if  $f(t_{\text{new}}) - \dot{f}(t_{\text{new}})\Delta t < 0$ , then A) if  $t_{\text{new}} \in (t_1, t_{\text{min}})$ , input  $(t_{\text{new}}, t_2)$  and  $f(t_{\text{new}})$ ,  $\dot{f}(t_2)$ ,  $\dot{f}(t_{\text{new}})$ , and  $\dot{f}(t_2)$  into step 1. End algorithm. However, B) if  $t_{\text{new}} \in (t_{\text{min}}, t_2)$ , then input  $(t_1, t_{\text{new}})$  and  $\dot{f}(t_1)$ ,  $\dot{f}(t_{\text{new}})$ ,  $\dot{f}(t_1)$ , and  $\dot{f}(t_{\text{new}})$  into step 1. End algorithm.

Before the roots are accepted as entry and exit times for that potential victim, the range from platform to satellite  $r_{\text{pv/s}}$  is compared to the maximum laser range  $\rho_{\text{max}}$ . If  $r_{\text{pv/s}} < \rho_{\text{max}}$ , then the roots are indeed entry and exit times; if not, then the potential victim does not enter the safety cone. Finally, the estimated entry and exit times are refined to a user-specified precision using the well-known Newton–Raphson root finding technique.<sup>3</sup>

## Results

A test case has been developed to verify the accuracy of the results as well as to give a measure of the time needed to run this analysis. The input parameters for this test case are as follows. The target satellite is catalog number 16609. The platform is located at 104°W longitude and 35°N latitude, with an elevation of 6350 ft. The cone half-angle was set to 5 deg. The test case begins 28 September 1995 at 16:07 GMT and has a lasing time of 4 min. Truth was derived for this case by propagating, with the same analytic propagator used in

Table 1 Safety cone entry and exit times<sup>a</sup>

Satellite number	Truth		Analysis	
	Entry	Exit	Entry	Exit
503	2.07	2.08	2.0688	2.079469
3431	2.91	3.28	2.907522	3.270386
5588	2.75	2.99	2.749378	2.986231
6058	2.19	2.33	2.181977	2.320286
7003	2.77	3.23	2.769862	3.226332
7052	0	0.13	0	0.127709
9862	2.74	3.05	2.738884	3.040841
9941	2.07	2.13	2.060931	2.129863
10155	3.02	3.15	3.013569	3.146149
10949	0.13	0.67	0.127479	0.663094
11079	0.55	0.58	0.545745	0.578429
12309	2.75	2.89	2.741594	2.882277
12472	2.76	3.05	2.750661	3.040511
12855	2.8	3.12	2.795004	3.119299
13012	1.36	1.55	1.352079	1.545441
13643	2.8	3.09	2.790403	3.080388
15098	2.86	3.18	2.856381	3.177383
15158	2.96	3.26	2.962593	3.25775
15272	3.59	4	3.580955	4
15383	2.92	3.19	2.919366	3.189581
15385	3	3.36	2.996685	3.352406
15487	2.77	3.05	2.763069	3.043048
15561	3.15	3.43	3.148435	3.426274
15642	3.05	3.17	3.045028	3.162653
15680	3.56	4	3.557147	4
15811	0.11	0.3	0.108985	0.296176
16187	1.02	1.26	1.016603	1.259595
16657	2.87	3.13	2.86822	3.127718
16925	1.06	1.31	1.053639	1.308601
17845	0	4	0	4
18578	2.85	3.2	2.846129	3.192031
18980	1.56	1.71	1.558795	1.701744
19215	2.91	3.27	2.901261	3.262419
19380	1.85	1.99	1.846384	1.980663
19440	1.07	1.24	1.069449	1.239369
19478	0.4	0.65	0.394882	0.647316
19592	3.34	3.73	3.334577	3.727426
19599	2.88	3.16	2.874781	3.15054
19608	1.82	1.93	1.819975	1.92806
19776	2.84	3.17	2.839927	3.160616
20028	1.33	1.47	1.32376	1.464188
20335	0	4	0	4
20433	0.84	1.08	0.833928	1.075884
20635	0	4	0	4
20926	2.96	3.34	2.954052	3.333315
21087	3.67	3.79	3.660626	3.783008
21216	1.06	1.32	1.050286	1.313879
21538	1.84	1.96	1.832421	1.956482
21900	2.69	2.97	1.234442	1.396503
21941	3.06	3.5	3.055651	3.495615
22057	0.99	1.26	0.986007	1.256211
22061	1.03	1.3	1.024112	1.292788
22176	3.03	3.46	3.024772	3.458724
22177	3.1	3.54	3.099429	3.533304
22192	2.02	2.09	2.010628	2.088627
22387	2.72	2.91	2.717225	2.903689
22671	1.88	2.03	1.873552	2.027885
22726	3.02	3.43	3.014858	3.422004
23027	1.5	1.65	1.498156	1.64331
23199	3	3.31	2.99789	3.302435
23452	3.14	3.59	3.136987	3.588506
23536	3.07	3.49	3.069929	3.488555
23579	0	4	0	4
23665	0	4	0	4

<sup>a</sup>Note that the analysis entry and exit times all fall within 0.01 min prior to the truth due to the manner in which truth was derived; that is, entry and exit criteria were checked at 0.01-min intervals to determine truth.

the analysis, in small time steps (0.01 min) and checking at each step if the potential victim lies within the safety cone. As seen in Table 1, every safety cone penetration listed in the truth is found by the analysis, and the entry and exit times match the truth for each satellite. Note that the entry time  $t_1$  listed in the truth actually indicates the satellite entered the safety cone at some time  $t$  satisfying  $(t_1 - 0.01 \text{ min}) \leq t \leq t_1$ . Likewise, the exit time  $t_2$  indicates a time  $t$  satisfying  $(t_2 - 0.01 \text{ min}) \leq t \leq t_2$ . Further, note that the

cone half-angle of 5 deg as well as the selection of the satellite in a crowded region result in the large number of safety cone penetrations. This test may not be realistic, but it provides a good test for finding many penetrations.

The analysis processed over 7000 potential victim satellites in less than 3.5 s on a laptop personal computer (266 MHz). It should be noted that these execution times do not include the propagation of the target throughout the period of interest. The assumption is that the target ephemeris, which in the test case is a list of 1000 positions and velocities, is either a given or can be calculated prior to the analysis.

At this point, it is reasonable to justify the inclusion of the filters and the spline technique by comparing execution times for the aforementioned test case. First, the analysis was run without the utilizing the filters or the splines. This procedure is essentially the same as the derivation of the truth results. For this analysis, the status (in or out of the safety cone) of the potential victim was checked at small time intervals (0.004 min) throughout the period of interest. The value of the small time increment was chosen for convenience while noting that it corresponds to less than half of the shortest period that a potential victim spends in the safety cone, as listed in Table 1. Notice that it is not clear a priori what the value of this time interval should be; thus, using this approach requires some preliminary decision about what minimum time a potential victim must spend in the safety cone before it will definitely be detected by the analysis. By using the spline technique, this decision is eliminated, and the analysis will find safety cone penetrations of arbitrarily small time periods. The analysis using neither the filters nor the spline executed in 936 s. This analysis did not use the Newton–Raphson root refining technique, and consequently, the precision of the entry and exit times was  $\pm 4.0 \times 10^{-3}$  min rather than  $\pm 1.0 \times 10^{-5}$  min specified in the Newton–Raphson procedure. Next, the test case was run using the filters but not the spline technique; the execution time decreased by nearly 50% to 486 s. As already mentioned, the execution time for the analysis incorporating both filters and spline technique was less than 3.5 s, which is less than 1% of the execution time using only the filters. These results are summarized in Table 2. The parameters used for the execution time comparisons are the same as though used for case 1, which are established in the next paragraph.

It is also interesting to examine the effectiveness of each filter individually. For this purpose, the parameters  $e_{low}$  and  $\rho_{max}$  are varied. Table 3 lists the five test cases including the original, case 1, which will be used for comparison. In case 2,  $\rho_{max}$  is reduced to 30,000 km, which is below the geosynchronous altitude. The minimum elevation angle  $e_{low}$  is varied in cases 3 and 4, and in case 5 both  $e_{low}$  and  $\rho_{max}$  are modified from their original values. The parameters not listed in Table 3 are identical to those used in case 1.

Table 4 lists the number of satellites eliminated by each filter in each test case as well as the number of satellites not eliminated by any filter, the number of safety cone penetrations, and the execution time. Note that each test case considered the same number of satellites. Because  $\rho_{max}$  has been reduced below geosynchronous altitude in case 2, the perigee filter, filter 3, becomes much more

**Table 4** Number of satellites<sup>a</sup> eliminated by each filter

Case	Filter number						Not filtered	Cone entries	Execution time, s
	1	2	3	4	5	6			
1	0	3	1	3731	408	587	2385	64	2.35
2	0	3	564	3511	403	582	2062	34	1.93
3	0	18	1	4527	430	557	1592	64	2.12
4	297	267	1	5492	276	401	391	28	1.48
5	0	18	564	4233	406	553	1351	34	1.67

<sup>a</sup>Total 7125.

effective and, consequently, reduces the execution time. Notice that the number of cone entries has also decreased; this is because the safety cone itself has been reduced in volume with the decrease in  $\rho_{max}$ . For case 3, there is a marginal increase in the effectiveness of the apogee filter, filter 2, and a corresponding decrease in execution time due to the reduction of the volume of the engagement envelope but not the safety cone. Notice that there are the same number of cone entries as in case 1. Note, however, that there are still no satellites that are filtered by the inclination filter, which is because with  $e_{low} = 30$  deg the engagement envelope for this particular platform still crosses the equatorial plane. To illustrate the utility of the inclination filter, case 4 increases  $e_{low}$  to 60 deg. The inclination filter and the apogee filter now are quite effective. Notice, however, that the number of cone entries has decreased from case 1 without any reduction in the volume of the safety cone because the engagement envelope has been reduced to an unrealistically small volume for this target scenario. That is, the engagement envelope has been shrunk to such an extent that the safety cone actually moves outside of it during the period of interest, which leads to erroneously filtered satellites and a subsequent reduction in the number of cone entries. Although unrealistic, case 4 demonstrates that for targets that do not have large angular motion relative to the platform, such as geosynchronous objects or calibration stars, the engagement envelope may be reduced to improve the effectiveness of the geometric filters. Last, case 5 demonstrates the combined effect of cases 2 and 3; notice that the execution time has decreased almost 30% from case 1.

The propagation tool used in the test cases is a modified version of the public release<sup>4,5</sup> of SGP4, the Air Force Space Command's analytic (also known as general perturbations) propagator. Note, however, that the filtering scheme is independent of the choice of propagator, and the spline technique is only marginally dependent on the type of propagator. That is, when using an analytic propagator, one can propagate the state to any point in time in one step. With a numerical or special perturbations propagator, the state is advanced through time in a series of small time steps. A typical time step for a low-Earth-orbit satellite using a moderate force model is 1 min. This time step is large compared to the time increments used to search for safety cone entries, but it does require multiple steps to propagate the state from the beginning to the end of periods of interest longer than 1 min. This calls for a modification of the spline technique such that the spline utilizes the state information available across the period of interest rather than just at the end points.

## Conclusions

Geometric filtering of potential victim satellites can increase the efficiency of the predictive avoidance analysis. The effect of this type of filtering is largely dependent on parameters such as platform location, laser range, and minimum elevation angle that govern the size and location of the engagement envelope. Although, for some geometries, these filters may not reduce the number of potential victims significantly, they are so computationally inexpensive that they do not affect the run time substantially. Furthermore, for those situations in which the geometric filters are effective, the execution time of the analysis may be significantly reduced. Nongeometric filters appear to reduce significantly the number of potential victim satellites necessary to consider in the safety cone analysis regardless of these parameters. The use of these filters will reduce the execution time of the analysis. These filters are valid for limited time periods; however, to consider longer periods of interest, multiple analyses may be run in series. The use of the cubic spline reduces the number of propagation calls necessary to determine entry and exit times

**Table 2** Execution time comparison

Filters	Spline	Newton–Raphson	Execution time, s
No	No	No	936
Yes	No	Yes	486
Yes	Yes	Yes	3.5

**Table 3** Test case parameters<sup>a</sup>

Test case number	$\rho_{max}$ , km	$e_{low}$ , deg	Platform parameters		
			Latitude, °N	Longitude, °W	Elevation, ft
1	40,000	15	35	104	6,350
2	30,000	15	35	104	6,350
3	40,000	30	35	104	6,350
4	40,000	60	35	104	6,350
5	30,000	30	35	104	6,350

<sup>a</sup>Latitude 35°N, longitude 104°W, and elevation 6350 ft.

within a prescribed precision. The spline allows for the estimation of a function that indicates whether a satellite is inside or outside of the safety cone. This estimation permits us to bypass the straightforward evaluation of the angle between satellites at small time intervals throughout the period of interest. Overall, the combination of filters and cubic splines appears to make a substantial improvement in the efficiency of the predictive avoidance analysis.

### Acknowledgments

The authors would like to express their appreciation to David Vallado, formerly of the U.S. Air Force Research Laboratory, for generously providing several astrodynamics routines including the modified version of SGP4. The authors would also like to thank Scott Wallace for several helpful discussions regarding the spline technique.

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A. C. Tribble  
Associate Editor