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EARTH ORIENTATION PARAMETER CONSIDERATIONS FOR PRECISE SPACECRAFT OPERATIONS

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Earth orientation parameters (EOP) are crucial for correctly converting between the Geocentric Celestial Reference Frame (GCRF) and International Terrestrial Reference Frame (ITRF), affecting parameters such as station coordinates, satellite positions, and non-spherical gravitational acceleration vectors. EOP interpolation methods and ocean tide corrections are shown to have a notable impact on these precise frame transformations. This paper investigates the accuracies and speeds of available methods that should be considered when using EOPs, allowing satellite operators and astrodynamicists to make informed decisions when choosing the best implementation for their individual needs.

INTRODUCTION

Satellite operations requiring ever-increasing position accuracy, such as Earth science missions, are on the rise. Additionally, software utilizing special perturbations for orbit propagation is rapidly replacing routines using general perturbations. These numerical models and precise position solutions require accurate frame transformations, particularly between the Geocentric Celestial Reference Frame (GCRF) and the International Terrestrial Reference Frame (ITRF) (*i.e.* inertial and fixed). The procedure and models needed to transform between the GCRF and ITRF are maintained by the International Earth Rotation Service (IERS).[1] Earth orientation parameters (EOP) are crucial for correctly executing these frame transformations, affecting parameters such as station coordinates, satellite positions, and non-spherical gravitational acceleration vectors. EOPs consist of the following:[1]

- Pole Coordinates (x_p,y_p): coordinates of the Celestial Intermediate Pole (CIP) with respect to the IERS Reference Pole (IRP) in the International Terrestrial Reference System (ITRS). The IRP is the location of the agreed upon terrestrial pole while the CIP is the actual axis of Earth rotation.
- Celestial Pole Offsets (dX,dY): observed corrections to the conventional celestial pole. The conventional celestial pole position is defined by the IAU Precession and Nutation models.
- UT1 Time Difference ($\Delta UT1$): the offset of Universal Time (UT1) from Universal Coordinated Time (UTC), *i.e.* $\Delta UT1 = UT1 - UTC$.

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- Length of Day (LOD): time difference between the observed duration of a mean solar day and 86,400 SI seconds.
- Atomic Time Offset (ΔAT): time difference between International Atomic Time (TAI) and Universal Coordinated Time (UTC), *i.e.* $\Delta AT = TAI - UTC$.

Although the Atomic Time offset is not technically an EOP, it is included in this list because this value is commonly incorporated into tabulated EOP data files. EOPs are published by the IERS once per day, effective at 0^{h} UTC of that day. These daily EOP values must be interpolated for maximum accuracy at other times during the day. In literature to date, however, recommended interpolation methods to achieve specific accuracies are not mentioned. Additionally, the published values of the pole coordinates, UT1 time difference, and length of day are "tide-free", meaning that users requiring high-precision need to apply additional corrections for diurnal and semi-diurnal ocean tides and libration after the parameter has been interpolated.[1] This fact is fairly unpublicized within astrodynamics literature.[2–4] This paper will present a comparison between the accuracy and speed of several commonly-used interpolation methods applied to EOPs during both frame transformations and orbit propagation. It will subsequently explore the effect of ocean tide corrections on the same items.

BACKGROUND

As mentioned, EOPs are needed to accurately transform from the ITRF to the GCRF and vise versa. Effective January 1, 2009 the IERS recommends the use of the new International Astronomical Union (IAU) models for nutation and precession to perform this GCRF/ITRF transformation.[1,5–10] The new model, as a whole, is called the IAU 2000A/2006 model. This includes the IAU 2000A nutation theory and the IAU 2006 precession theory. Several methods for implementing this new model have been developed over the past few years, but are all within the uncertainty of the IAU model itself.[2,8,9,11] The CIO-based (Celestial intermediate origin) series method of the IAU 2000A/2006 model is utilized to perform the GCRF/ITRF transformations for this paper. Equations (1–5) below describe the general process of this transformation and more details on the IAU model and implementation processes can be found in References 1, 2 and 11.

$$\left[\mathbf{W}\right] = \mathrm{ROT3}(-s')\mathrm{ROT2}(x_p)\mathrm{ROT1}(y_p) \tag{1}$$

$$\left[\mathbf{R}\right] = \mathrm{ROT3}(-\theta_{ERA}) \tag{2}$$

$$\left[\mathbf{PN}\right] = \begin{bmatrix} 1 - aX^2 & -aXY & X \\ -aXY & 1 - aY^2 & Y \\ -X & -Y & 1 - a(X^2 + Y^2) \end{bmatrix} \mathrm{ROT3}(s) \tag{3}$$

$$\vec{r}_{GCRF} = \left[\mathbf{PN}\right]\left[\mathbf{R}\right]\left[\mathbf{W}\right]\vec{r}_{ITRF} \tag{4}$$

$$\vec{v}_{GCRF} = \left[\mathbf{PN}\right]\left[\mathbf{R}\right]\left\{\left[\mathbf{W}\right]\vec{v}_{ITRF} + \vec{\omega}_{\oplus} \times \left[\mathbf{W}\right]\vec{r}_{ITRF}\right\} \tag{5}$$

Figure 1 below identifies which EOP contributes to each of the components needed for the frame transformation. To be in full compliance with the models of the IERS Conventions (2010), namely the IAU 2000A/2006 model, the pole coordinates, UT1 time difference, and length of day must first be interpolated to the appropriate time and then modified by ocean tide corrections and libration corrections. This is because the subdaily variations are not part of the values posted by the IERS. Equation (6) below shows the naming convention for each parameter.[1]

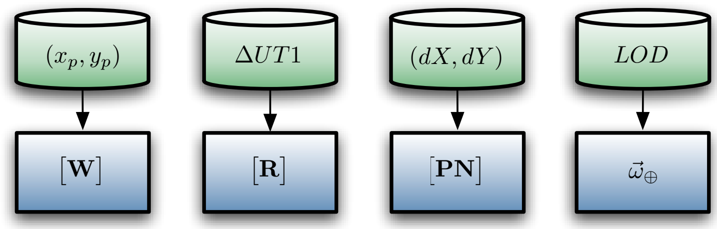

Figure 1. Depiction of which EOP is used to generate each element in the transformation process.

$$(x_p, y_p) = (x_p, y_p)_{\text{IERS}} + (\Delta x, \Delta y)_{\text{ocean tides}} \tag{6a}$$

$$\Delta UT1 = \Delta UT1_{\text{IERS}} + \Delta UT1_{\text{ocean tides}} \tag{6b}$$

$$LOD = LOD_{\text{IERS}} + \Delta LOD_{\text{ocean tides}} \tag{6c}$$

The libration corrections, $(\Delta x, \Delta y)_{\text{libration}}$, are not shown in Eq. (6) and are not included in this investigation for two reasons: 1) they are an order of magnitude smaller than the ocean tide corrections and 2) the model has, to the best of the authors' knowledge, not been verified by observations as of yet. These libration corrections consist of diurnal and semi-diurnal nutations due to external torque, namely luni-solar torque.[1] The corrections, $(\Delta x, \Delta y)_{\text{ocean tides}}$, include diurnal and semi-diurnal variations caused by ocean tides for the three EOPs in Eq. (6).[1]

The subscript "IERS" indicates the daily published values that have been interpolated. These parameters are published from several sources, including the IERS, National Geospatial-Intelligence Agency (NGA), and the United States Naval Observatory (USNO), however CelesTrak has compiled EOPs from all available sources to create a single file containing EOP data from 1962 up to predictions 1 year into the future.* The file maintained by CelesTrak is used by Analytical Graphics Inc. Satellite Tool Kit (STK).[4]

The process of interpolating EOPs and adding corrections is outlined by the flowchart shown in Figure 2. It is prudent to remember that the first step is to interpolate any EOP of interest. The computation of the ocean tide corrections is relatively simple and only requires an input of Julian Centuries of Terrestrial Time, T_{TT}. The IERS Conventions (2010) provide the algorithm for computing the ocean tide corrections using a table of amplitudes and arguments for 71 tidal components and FORTRAN code is available online from the IERS EOP Product Center.[1] Two different source code files named `INTERP.F` and `ORTHO_EOP.F` are available online for the computation of ocean tide corrections.†‡ The file `ORTHO_EOP.F` is based on the full model from Reference 12 and the file `INTERP.F` uses the table of 71 tidal constituents derived from `ORTHO_EOP.F`. The two routines agree within a few microarcseconds in polar motion and a few tenths of a microsecond in UT1.[1] The investigation reported in this paper used the same method as `INTERP.F`.

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* http://www.celestrak.com/SpaceData/

† `INTERP.F` available here: ftp://hpiers.obspm.fr/eop-pc/models/interp.f

‡ `ORTHO_EOP.F` available here: ftp://tai.bipm.org/iers/conv2010/chapter8/ORTHO_EOP.F

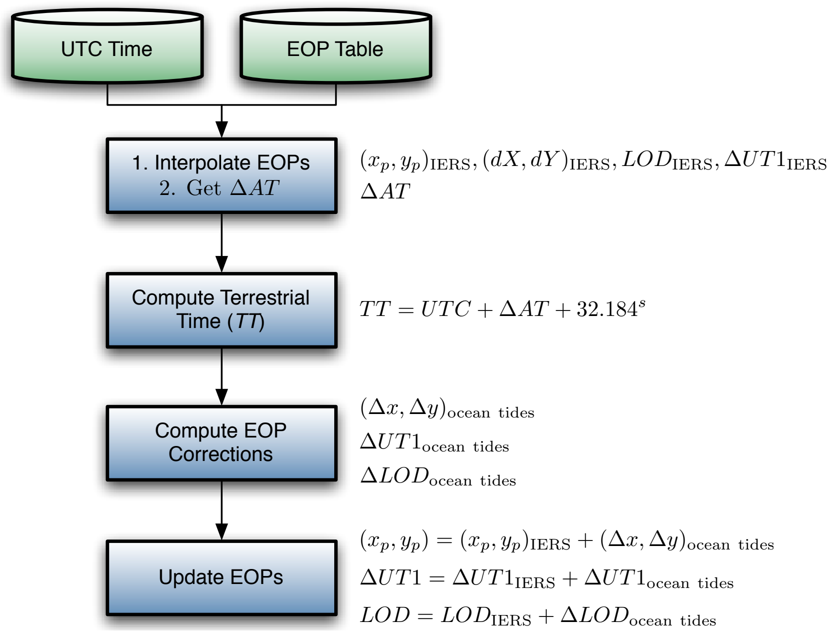

Figure 2. Flowchart of EOP interpolation and ocean tide correction process.

The `INTERP.F` routine, mentioned above, also incorporates EOP interpolation into its procedure. The default is a 4-point window Lagrangian interpolation (*i.e.* 4^{th}-order Lagrange). Although this is the default interpolation method, it does not mean it is the most accurate. Reference 3 suggests that using a quadratic polynomial interpolation for the pole coordinates and $\Delta UT1$ will yield sufficient accuracy and Reference 13 suggests that linear interpolation is a common standard.

Before results are presented it is beneficial to know how the EOPs and individual ocean tide corrections behave over time. Figure 3 displays the daily EOP values for all six parameters over 3 months. It is immediately clear that the length of day and celestial pole offsets contain the most dramatic variation over time. The pole coordinates and time offset parameters, however, exhibit a much smoother variation over time, something to keep in mind during the comparison of interpolation methods.

As shown in Figure 4 below, corrections are only applied to the pole coordinates, $UT1$ time offset, and length of day parameters. The celestial pole offsets are *observed* corrections to the precession-nutation theory. The corrections are oscillatory as would be expected from ocean tides and are quite distinct. The time window in Figure 4 has been reduced to one day to clearly show the effect of the diurnal and semi-diurnal ocean tide corrections. The corrections to $\Delta UT1$ have a RMS amplitude of about 25 μs and reach as high as 73 μs in amplitude during this 3-month window of time.

It should be noted that high-rate EOP estimation (sub-daily) is another field of research that is ongoing. The models described and used here are constantly in the process of being verified and improved by research groups performing EOP estimation from GPS and VLBI measurements.[13–15] It is important to be mindful of the accuracy of each model and keep track of progress being made on these fronts, because it directly impacts precise frame transformations, as shown in this paper.

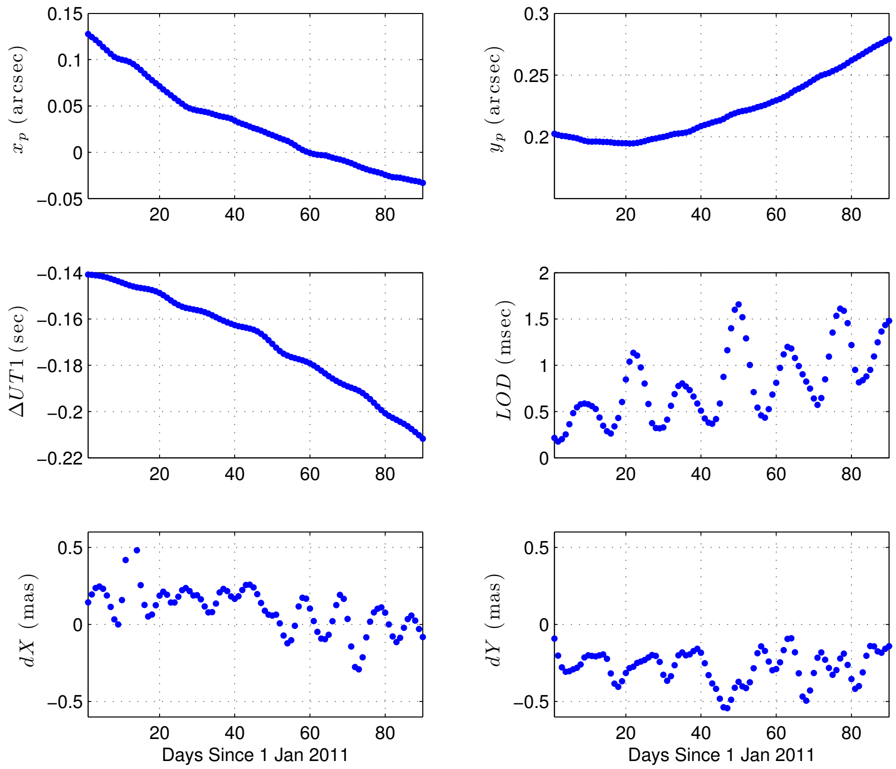

Figure 3. Daily tabulated EOPs over the 3-month period of investigation.

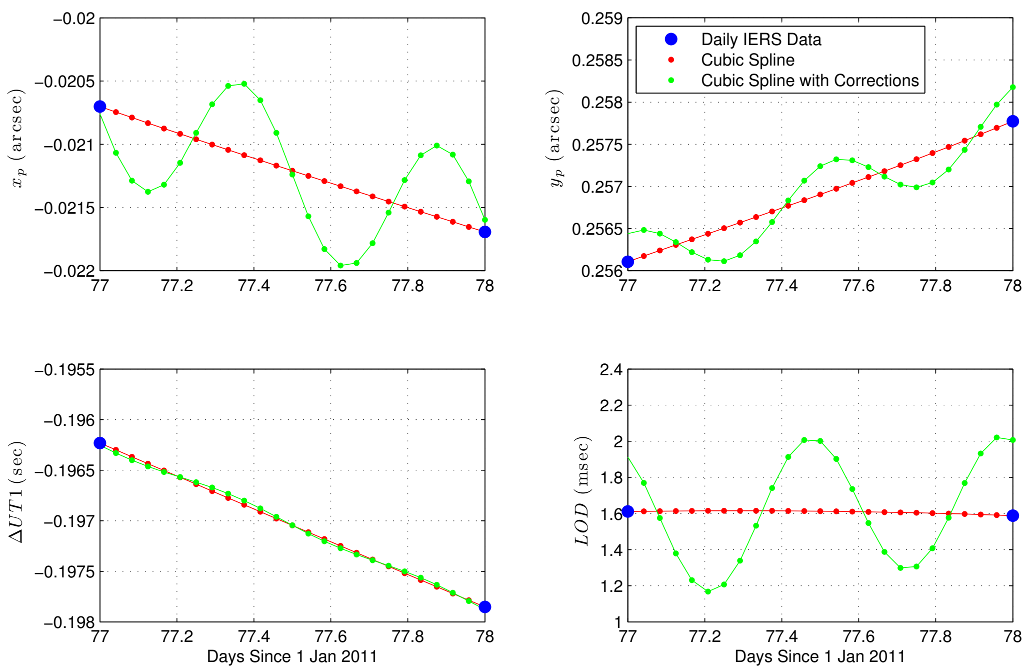

Figure 4. Ocean tide corrections for applicable EOPs on 19 Mar 2011.

METHODS

This section details the steps taken to investigate position frame transformations between the ITRF and GCRF as well as orbit propagation. For each of the two investigations, truth is considered to be the use of a cubic spline to interpolate all EOPs. The geophysical quantities used here, such as the location of Earth's rotation axis and the spin rate, vary continuously with time. Cubic spline interpolation provides these continuous and smooth properties inherent in these geophysical motions. The addition of ocean tide corrections will be investigated separately. Several interpolation methods as well as the exclusion of various EOPs are studied. Excluding individual EOPs will provide insight into the importance of each EOP in the frame transformation process. Each method investigated in this paper is listed below. Last on the list is a method of transforming between inertial and fixed frames using a rotation of Greenwich Mean Sidereal Time (GMST) only. This simple rotation ignores nutation, precession, and polar motion and is commonly used in university simulation studies.

- Cubic spline interpolation with and without ocean tide corrections
- Linear interpolation and 3$^{\mathrm{rd}}$, 5$^{\mathrm{th}}$, 7$^{\mathrm{th}}$, and 9$^{\mathrm{th}}$-order Lagrange interpolation
- $(x_p, y_p) = 0$, and cubic spline on the rest
- $\Delta UT1 = 0$, and cubic spline on the rest
- $(dX, dY) = 0$, and cubic spline on the rest
- All EOPs equal to zero
- Simple GMST rotation only (UTC time used)

Frame Transformation

A grid of points was generated at several altitudes to explore the effect that the methods described above have on position frame transformations. Four altitudes typical for ground stations, science satellites, GPS satellites, and communication satellites were used and are listed in Figure 5 below. The four altitudes correspond to the Earth's surface (modeled here by the WGS84 ellipsoid), low-Earth orbits (LEO), Global Positioning System (GPS) orbits, and geosynchronous orbits (GEO).

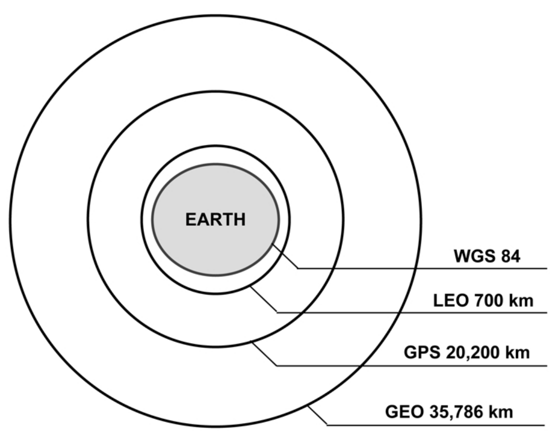

Figure 5. Altitudes used for position frame transformations.

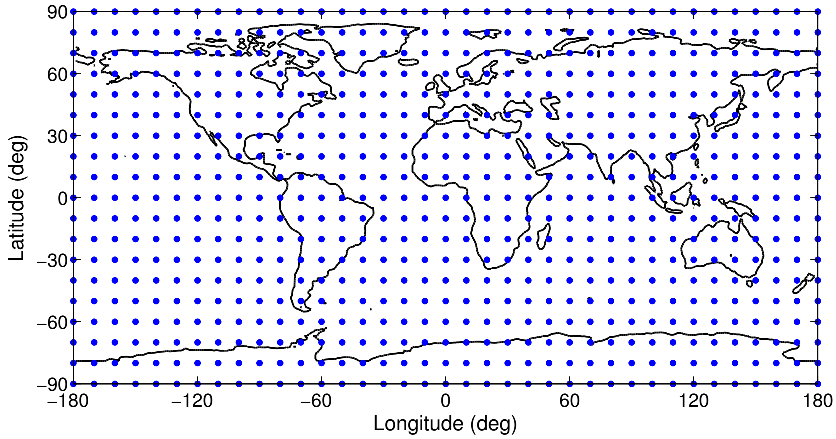

Figure 6. Grid points used for position frame transformations.

The grid is composed of points every 10 degrees in longitude and latitude in order to achieve high spatial resolution and global coverage. Each position is transformed from the ITRF to the GCRF every 1 hour from 0^h January 1st, 2011 to 0^h April 1st, 2011 UTC. This 3-month time span with 1-hour steps yields excellent temporal resolution, well above the Nyquist rate for the ocean tide corrections.

Orbit Propagation

During orbit propagation using special perturbation techniques, EOPs contribute to the computation of non-spherical gravitational acceleration vectors, the determination of Earth's instantaneous spin axis, as well as the current rotation rate of the Earth. The spin axis and rotation rate affect atmospheric drag calculations, however, are not investigated in this paper. The effect that EOPs have on gravitational acceleration vectors is investigated. To compute the acceleration due to a gravity field, such as one described by spherical harmonics, the position of the satellite must first be rotated from the inertial frame (GCRF) to the fixed frame (ITRF). The ITRF position vector is then used to compute the gravitational acceleration vector in the ITRF frame. In order to numerically integrate this acceleration, the ITRF acceleration vector must be rotated to the GCRF. Thus, EOPs are used in two frame transformations for every computation of a gravitational acceleration vector.

Three orbit types were used to explore the effect of EOPs on orbit propagation: LEO, GPS, and GEO. Tables 1 and 2 show the initial conditions for each orbit as position and velocity vectors and as osculating classical orbital elements. Each orbit was propagated for 7 days, starting at 0^h on January 1st, 2011 using a 50x50 GGM02C gravity field. A fixed-step Runge-Kutta-Fehlberg 7^{th}-order integrator was used to propagate the LEO, GPS, and GEO orbits with step sizes of 5, 20, and 60 seconds, respectively.

Table 1. Initial state vectors in GCRF frame of each orbit investigated.

Name	x (m)	y (m)	z (m)	v_x (m/s)	v_y (m/s)	v_z (m/s)
LEO	6,715,724.634	105,595.622	-336,183.121	123.035	6,319.490	4,400.611
GPS	-25,710,512.141	-3,287,462.745	-4,499,975.080	820.822	-2,255.680	-3,087.641
GEO	32,455,558.698	26,849,573.028	1566.130	-1,961.727	2,371.514	0.525

Table 2. Initial osculating classical orbital elements of each orbit investigated.

Name	a (m)	e	i (deg)	Ω (deg)	ω (deg)	ν (deg)
LEO	6,730,038.573	0.0008023	35.00002	4.99999	335.04742	19.95260
GPS	26,559,650.932	0.0097005	53.84996	0.0	180.20264	12.02735
GEO	42,164,118.245	0.0009997	0.01000	27.30363	9.99757	2.29880

RESULTS AND DISCUSSION

Position Frame Transformation

Table 3 shows the maximum 3D position errors seen over the 3-month window for each interpolation method. As expected, the frame transformation error grows with altitude since transformation errors stem from differences in the central angle. Linear interpolation of the EOPs produces the worst errors compared to all Lagrange interpolation methods investigated while 9$^{\rm th}$-order Lagrange interpolation produces the smallest errors. Even so, linear interpolation only results in errors around 2 cm on Earth's surface and at LEO altitude. Depending on the application, this magnitude of error could be quite acceptable. For Lagrange interpolation, as the order increases from 3 to 9, errors decrease from about 5-6 mm to less than 1 mm for locations on the Earth's surface and at LEO altitude.

Table 3. Maximum 3D position errors of each interpolation method compared to cubic spline. Errors are expressed in meters.

Interpolation Method	Surface	LEO	GPS	GEO
Linear	0.0186	0.0206	0.0773	0.1227
Lagrange 3$^{\rm rd}$	0.0053	0.0059	0.0222	0.0352
Lagrange 5$^{\rm th}$	0.0017	0.0019	0.0070	0.0111
Lagrange 7$^{\rm th}$	0.0010	0.0011	0.0043	0.0068
Lagrange 9$^{\rm th}$	0.0008	0.0009	0.0032	0.0052

The effect of ocean tide corrections is shown in Table 4 below. The position frame transformation error can be up to several centimeters on the surface and in LEO and up to nearly 20 cm in GEO. Again, depending on the application, ocean tide corrections could be ignored or absolutely necessary to include, especially since some precise orbit determination applications require position accuracies down to centimeter and even millimeter levels.

Table 4. Maximum 3D position errors of cubic spline interpolation versus cubic spline with ocean tide corrections applied. Errors are expressed in meters.

Interpolation Method	Surface	LEO	GPS	GEO
Cubic Spline	0.0264	0.0294	0.1669	0.1750

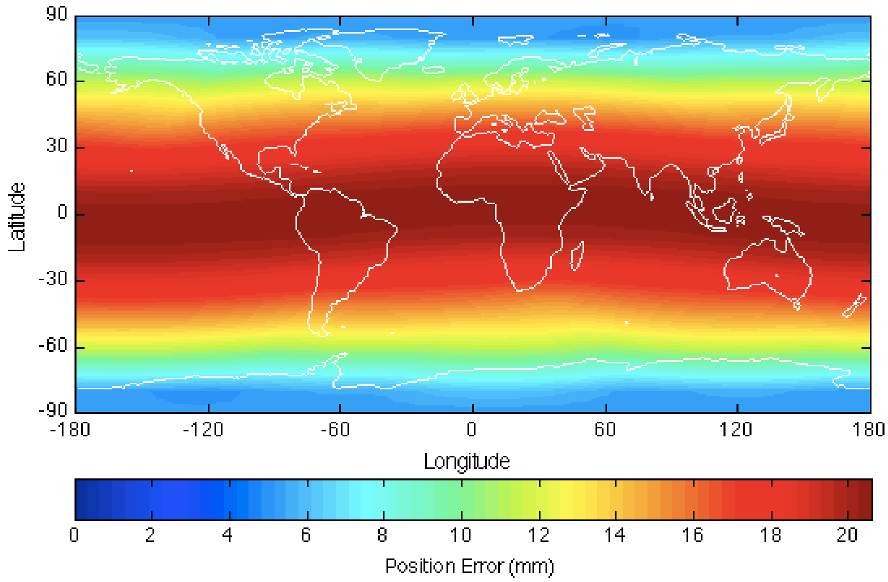

Figure 7. Position error between cubic spline and linear interpolation at LEO. Errors represent maximum error at each point for the entire time window.

Figure 7 is an example of the distribution of position errors comparing linear interpolation to the use of a cubic spline in LEO. The largest errors occur near the equator, decreasing as positions near the poles. This is to be expected since equatorial points are farther away, radially, from the axis of rotation than points at higher latitudes.

Results from the additional study excluding certain EOPs are displayed in Table 5. The celestial pole offsets contribute the smallest amount to the frame transformation as shown below. These errors are similar to the ocean tide corrections in magnitude, thus allowing for the possibility of exclusion in cases not requiring the highest accuracy. The effect of the pole coordinates and $UT1$ offset are quite large, however. The $\Delta UT1$ parameter clearly has the largest effect, contributing to 99.6% of the error when compared to ignoring EOPs entirely.

Table 5. Maximum 3D position errors of each EOP set compared to using all EOPs with a cubic spline interpolation. Errors are expressed in meters.

EOP Set	Surface	LEO	GPS	GEO
$(dX, dY) = 0$	0.0214	0.0238	0.0893	0.1416
$(x_p, y_p) = 0$	8.6790	9.6469	36.2239	57.4664
$\Delta UT1 = 0$	98.4674	109.2742	410.3204	650.9412
No EOPs	98.8495	109.6982	411.9128	653.4674
GMST Rotation only	17,662.4676	19,601.6792	73,603.5648	116,766.3028

The last item in Table 5 is a case using a simple rotation through Greenwich Mean Sidereal Time (GMST) only, thus ignoring EOPs and the IAU 2000A/2006 nutation-precession model entirely. This method results in tens of kilometers of error in position at LEO altitude and over 100 km of error at GEO altitude. Clearly, this procedure should only be used if coarse positions are required, such as when performing visibility calculations.

Orbit Propagation

Orbit propagation is shown to be quite unaffected by choice of EOP interpolation method. Table 6 reveals that orbit propagation errors are below 2 mm over 7 days for each orbit and for every interpolation method discussed. In fact, the GPS and GEO orbits exhibit errors well into the sub-millimeter range. Again, the errors displayed below are the maximum 3D position differences in propagation over a 7-day integration period compared to a propagation using cubic spline interpolation for all EOPs.

Table 6. Maximum 3D position errors of each interpolation method compared to cubic spline during orbit propagation. Errors are expressed in meters.

Interpolation Method	LEO	GPS	GEO
Linear	0.0004	0.0000	0.0000
Lagrange 3^{rd}	0.0020	0.0001	0.0000
Lagrange 5^{th}	0.0020	0.0000	0.0000
Lagrange 7^{th}	0.0019	0.0001	0.0000
Lagrange 9^{th}	0.0015	0.0000	0.0000

There are two additional items worth noting in the Table 6 results. First, linear interpolation resulted in a smaller maximum error during propagation than any of the Lagrange methods. This is a product of the fact that this is one orbit, with one initial state vector, and one initial time, thus being a random case when linear interpolation happens to result in smaller errors. Considering the propagation lasts for seven days, the errors are all quite small and sub-millimeter differences are not necessarily meaningful. The second item is that the propagation error decreases with altitude, which is opposite from what is seen with the position frame transformation errors. This is due to two factors: 1) higher altitudes are less sensitive to higher degree gravity terms and thus less sensitive to small errors in the gravitational acceleration vector, and 2) the smaller time step of the LEO orbit means that errors in the acceleration vector occur many more times than during the GEO propagation.

Table 7. Maximum 3D position errors of cubic spline interpolation versus cubic spline with ocean tide corrections applied during orbit propagation. Errors are expressed in meters.

Interpolation Method	LEO	GPS	GEO
Cubic Spline	0.1470	0.0003	0.0004

Table 7 shows that the effect of incorporating ocean tide corrections is also negligible for GPS and GEO altitude orbits. The effect of tide corrections on the LEO orbit is less than 15 cm after 7 days. Although EOP interpolation techniques have a minimal effect for most orbit propagation applications, excluding certain EOPs entirely can have a measurable impact. Ignoring all EOPs is shown to cause a difference in orbit propagation of almost 10 meters for the LEO case investigated here. Once the orbital altitude reaches GPS and GEO altitudes, however, EOPs have a much smaller influence. The use of the simple GMST rotation to transform the gravitational acceleration vector once again results in poor accuracy. Errors are greater than 7 km in LEO and even approach 100 m in GEO, verifying that the use of GMST rotation alone should only be implemented when coarse propagations are acceptable.

Table 8. Maximum 3D position errors of each interpolation method compared to cubic spline during orbit propagation. Errors are expressed in meters.

Interpolation Method	LEO	GPS	GEO
$(dX, dY) = 0$	0.0241	0.0006	0.0001
$(x_p, y_p) = 0$	8.7107	0.1245	0.0092
$\Delta UT1 = 0$	0.6714	0.0348	0.0439
No EOPs	8.0413	0.1539	0.0373
GMST Rotation only	7,575.0655	289.7671	71.9633

Interpolation Speed Comparison

The speed of each interpolation method is an important factor when choosing the best implementation for the given task. Table 9 displays normalized timing results from a standard dual core 2 GHz laptop computer. Each processing time is reported as relative time compared to the results of linear interpolation and as speed-up factors compared to the use of a cubic spline with ocean tide corrections applied. A table of EOPs was interpolated every 2 minutes for 3 months for each interpolation method and repeated 10 times to ensure fairly accurate timing results.

Table 9. Normalized time required to interpolate EOPs every 2 minutes for 3 months. Relative timing is given with respect to linear interpolation and speed-up factors are given with respect to the use of cubic spline interpolation with ocean tide corrections.

Interpolation Method	Relative Timing	Speed-up Factor
Linear	1.00	30.44
Lagrange 3^{rd}	1.14	26.59
Lagrange 5^{th}	1.17	25.95
Lagrange 7^{th}	1.20	25.39
Lagrange 9^{th}	1.23	24.64
Lagrange 9^{th} + Tides	2.91	10.47
Cubic Spline	28.48	1.07
Cubic Spline + Tides	30.44	1.00

Linear interpolation is clearly the fastest, however, has been shown to be less accurate than the Lagrange methods when transforming positions. As the order of Lagrange interpolation increases, the relative time required increases only slightly, making 9^{th}-order Lagrange interpolation very attractive. Ninth-order Lagrange is both fast and close in accuracy when compared to the cubic spline.

CONCLUSION AND RECOMMENDATIONS

Interpolation methods, as well as ocean tide corrections, applied to EOPs are shown to have a notable impact on position frame transformations between fixed (ITRF) and inertial (GCRF) frames. The effect of individual EOPs being excluded from the implementation has also been presented. These results demonstrate the need for orbital analysts to carefully consider EOP implementation strategies based on the given task, including orbital altitude and desired position accuracy. The table below provides recommendations for the use of EOPs, interpolation methods, and the inclusion of ocean tide corrections to achieve various levels of position accuracy. Although this investigation limited its analysis to four altitude regimes, these recommendations are thought to be very good estimates of achievable accuracies. Table 10 tries to encompass accuracies for surface and GEO altitudes so the Results section of this paper should be consulted for specific errors encountered in each altitude region. While the use of a cubic spline and ocean tide corrections has been considered to be the highest accuracy option, it is still given a 1 mm accuracy level in the recommendation table below in an effort to acknowledge small imperfections and limitations in IAU models, EOP observations, and in the ocean tide model.

Table 10. Recommended uses of EOPs to achieve various levels of position frame transformation accuracy while using the fastest procedure possible. For all accuracy levels with a range of values the lower bound is for the Earth's surface and the upper bound is for GEO altitude.

Desired Accuracy Level	EOPs	Interpolation	Ocean Tides
20 km – 120 km	none (GMST)	–	–
100 m – 1 km	none	–	–
10 m –100 m	$\Delta UT1$	Linear	No
4 – 30 cm	$(x_p, y_p), \Delta UT1$	Linear	No
3 – 20 cm	All EOPs	Linear	No
1 – 5 mm	All EOPs	Lagrange 9th	Yes
$<$ 1 mm	All EOPs	Cubic Spline	Yes

Furthermore, we conclude that EOP interpolation methods and ocean tide corrections are insignificant for orbit propagation at any altitude region for most applications. The use of linear, 9th-order Lagrange, or cubic spline interpolation result in orbit propagations identical down to the millimeter level over 7 days. This indicates that orbit propagation is robust at handling errors in EOP predictions, making future orbit propagations reliable in this sense, an important quality for space situational awareness and orbit debris propagation.

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